Application of Dipole Density versus Voltage in Electrophysiology: A Model Study

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Abstract

Background:
Voltage measured with direct contact is believed to be the best measure of cardiac activation. However, voltage is a far-reaching summation of local positive and negative sources of charge that propagate across the myocardium from the action of ion channels. It is clinically desirable to measure these local sources.

Methods:
Local sources were modeled as a double-layer of dipole density (DD) on the endocardial surface (ES) with orientation of the dipoles perpendicular to the ES. A sphere (radius=1) was used to model the ES and the function exp(-αcosθ) to model DD on the ES (DD=1 at the South Pole of the ES and exponentially decreasing to 0 at the North Pole). Exact voltage can be calculated everywhere for this model. Voltage was calculated across the ES and at 186 points on a non- contact measurement sphere (radius=0.5) centered within the ES and applied in a DD inverse algorithm that reconstructed DD on the ES. Reconstructed DD was compared to the exact model of DD.

Results:
The voltage has a long rightward tail, compared to the finite range of dipole density. The RMS error between the reconstructed DD and the exact model of DD was 0.01 (1%) across the entire ES.

Conclusions:
This model demonstrates the basic feasibility of inverse reconstruction of DD on the ES from an array of non-contact measurements of V and opens the possibility of high resolution global mapping without the challenges of maintaining direct contact. Further study on human anatomy with biological sources is warranted.

Background

The voltage at any location on the endocardial surface can be separated into two components: local dipole source and distant sources (Figure 1).

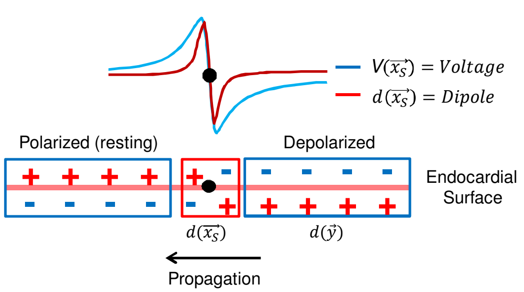

Figure 1: Local dipole density source $d(\vec{x_S})$ and voltage $V(\vec{x_S})$ during depolarization.

The following equation describes the relationship between dipole density and voltage on the surface:

$$\underbrace{V(\vec{x}_S)}_{\text{Voltage at any point}} = \underbrace{-2\pi d(\vec{x}_S)}_{\text{Local source Dipole Density}} + \underbrace{\int_S d(\vec{y})\frac{\cos\varphi_{xy}}{|\vec{x}-\vec{y}|^2}\, dS_y}_{\text{Sum of distant sources (surrounding voltage)}}$$

φ_{xy} = Angle betwen vetcor $\vec{x}-\vec{y}$ and the normal $\vec{n}$
$d(\vec{x_S})$ = Dipole density at $\vec{x_S}$ on the surface
S = Surface
$V(\vec{x_S})$ = Voltage at $\vec{x_S}$ on the surface

Ref: arxiv.org/abs/1006.3453: Electrophysiology Of Living Organs From First Principles

Methods

1. The dipole density distribution on the epicardial surface (radius = 1) is modeled by the function: exp(-αcosθ).
2, The voltage can be calculated exactly on 186 points inside of the surface (radius = 0.5), representing a multiple electrodes catheter (Figure 2).

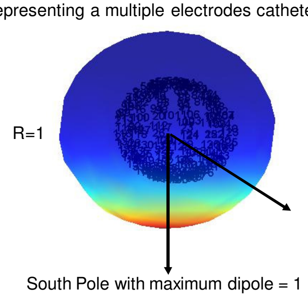

Figure 2: Dipole density distribution on the sphere unit and 186 voltage position inside of the surface (r=0.5).

3. An inverse algorithm is used to reconstruct the dipole density on the epicardial surface based on the measured voltage at the 186 points.

$$d_{rec} = W^{-1} * V$$

d_{rec} = Reconstructed dipole density
W = Transfert matrix
V = Voltage

The inverse algorithm requires regularization because the problem is mathematically ill-posed.

4. These results are compared to the theoretical dipole density value and the RMS error is calculated.

Results

The comparison between the voltage and the dipole density shows a long tail of the voltage in contrast to the localized dipole density (Figure 3A).
The RMS error between the reconstructed dipole density and the exact model was 0.01 (1%) across the entire ES (Figure 3B).

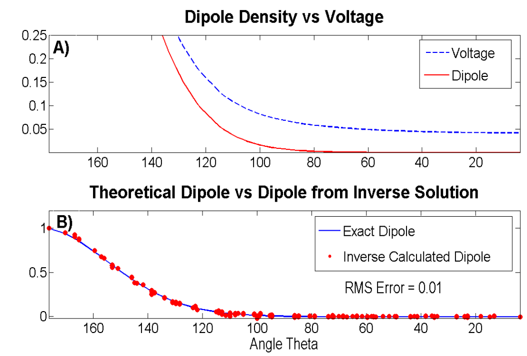

Figure 3: A) Dipole Density vs Voltage. B). Inverse Solution

Conclusion

The method is currently under clinical investigation and first results demonstrated the validity and the superiority of the dipole density over the voltage maps (Figure 4).

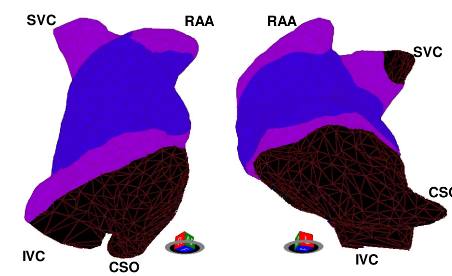

Figure 4: Right atrium: Voltage (violet) vs Dipole Density (blue) maps

Declaration of Interest

Lam Dang: consulting fees/stock options from Acutus Medical Inc. Research grants from Biosense Webster, Inc.
Graydon Beatty: salary/stock options from Acutus Medical Inc.
Günter Scharf: consulting fees from Acutus Medical Inc.